A Novel Application of the Banach Contraction Principle: Liquidity Adjusted Stress Propagation and Equilibrium in Financial Networks

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ABSTRACT

We develop a mathematical fixed-point framework to study stress propagation in financial networks that combines interbank exposures and liquidity-induced market impact. By defining a stress-propagation operator on the space of institution-level stress vectors, we derive a sufficient, economically interpretable inequality that guarantees the operator is a contraction in the supremum norm. Through the Banach fixed-point theorem, we obtain rigorous existence, uniqueness, and geometric convergence results for the systemic-stress equilibrium. Explicit constants, detailed proofs, sensitivity analysis, and two real-world style applications a small banking contagion network and a portfolio-liquidity feedback model are presented. Numerical illustrations confirm convergence and provide insights into systemic stability under liquidity stress.

Keywords: Banach contraction principle; fixed point; financial networks; liquidity risk; market impact; systemic stability.

1. Introduction

In modern financial systems, interdependence among institutions has created intricate channels through which distress at one entity can spread throughout the entire network. The collapse of a single large participant may propagate losses via direct credit exposures, common asset holdings, or liquidity shocks, producing system-wide contagion and instability. The mathematical understanding of such phenomena collectively referred to as systemic risk has therefore become a central topic in quantitative finance, risk management, and financial regulation.

Classical contagion models, beginning with the interbank frameworks of Allen and Gale (2000), interpret systemic crises as cascades of defaults driven by bilateral obligations. Subsequent works such as Cifuentes, Ferrucci, and Shin (2005) incorporated liquidity and market-impact effects, highlighting that even solvent institutions may become distressed when forced to liquidate assets in illiquid markets. These contributions established that feedback loops between market prices and balance-sheet constraints can generate multiple equilibria, discontinuous transitions, and complex nonlinear behavior. While these models provide valuable intuition, most rely on iterative numerical simulations or spectral conditions that offer limited analytical guarantees regarding existence, uniqueness, or convergence of equilibria.

From a mathematical perspective, these equilibrium problems naturally invite an operator theoretic treatment. The Banach Contraction Principle, one of the cornerstones of nonlinear functional analysis, offers a precise and elegant tool to establish existence and uniqueness of fixed points for self-mappings on complete metric spaces. Originally formulated by Stefan Banach in 1922 (Banach, 1922), the principle has since permeated diverse disciplines from differential equations and optimization to dynamic programming (Blackwell, 1965; Deonardo, 1967). Despite its fundamental nature, its direct application to financial-network equilibrium analysis has remained largely unexplored. Most financial contagion models rely instead on spectral-radius or monotonicity conditions that do not exploit the geometric convergence guarantees available through contraction mappings.

The objective of this paper is to bridge this gap by constructing a mathematically rigorous connection between the Banach Contraction Principle and the nonlinear stress-propagation dynamics of financial networks. We develop an explicit operator, termed the liquidity-adjusted stress-propagation operator, that updates institution-specific stress levels through two interdependent mechanisms: (1) linear contagion from interbank exposures, and (2) nonlinear liquidity amplification arising from market-impact feedbacks. We show that when these mechanisms satisfy economically interpretable stability constraints specifically, when the combined effect of counterparty and liquidity amplification remains below unity the mapping is a strict contraction under the supremum norm. This immediately guarantees the existence of a unique systemic-stress equilibrium and the geometric convergence of simple iterative algorithms used to compute it.

The novelty of this framework is twofold. First, it translates intricate financial feedback processes into a compact mathematical form amenable to fixed-point analysis. Second, it establishes an explicit and verifiable inequality connecting observable economic quantities such as interbank leverage, liquidity sensitivity, and market-impact curvature to the contraction constant that governs equilibrium stability. This formulation converts qualitative notions of “financial fragility” into quantitative stability thresholds with rigorous convergence guarantees.

The results obtained here complement and extend recent work on network stability and systemic risk (Acemoglu et. al., 2015; Battiston et. al., 2012). Unlike spectral approaches that linearize contagion near equilibrium, the present method accommodates fully nonlinear liquidity feedbacks through sublinear market-impact functions. Moreover, the approach provides direct bounds on the rate of convergence of iterative stress-testing algorithms of practical relevance to regulators and central banks that implement recursive stress-propagation computations.

The remainder of the paper is organized as follows. Section 2 introduces the mathematical model, specifying the structure of interbank exposures, liquidity sensitivities, and market-impact functions. Section 3 formulates the stress-propagation operator and presents the main contraction theorem, establishing conditions for existence, uniqueness, and geometric convergence of the equilibrium. Section 4 develops explicit closed-form calculations in stylized settings to illustrate the theorem. Sections 5 and 6 present two real-life examples a banking contagion network and a portfolio-liquidity feedback model followed by interpretations and policy implications. The final sections discuss extensions, empirical calibration, and concluding remarks.

2. Model and notation

Let n denote the number of institutions. Stress levels are represented by a vector $x = (x_1, \ldots, x_n)^\top \in [0, \overline{x}]^n$, where $\overline{x} > 0$ is an upper bound (normalization). $E = (e_{ij}) \in \mathbb{R}_+^{n \times n}$. Let be the nonnegative exposure matrix where e_{ij} denotes normalized exposure of institution $i \; to \; j$.

2.1 Liquidity and market-impact structure: Each institution $\boldsymbol{i}$ generates liquidation volume responsive to its own stress and that of others:

$$\ell i(x) = \lambda_i x_i + \sum_{j=1}^{n} \gamma_{ij} x_j, \qquad \lambda_i, \gamma_{ij} \geq 0. \tag{2.1}$$

Aggregate liquidation is

$$L(x) = \sum_{k=1}^{n} \ell_k(x). \tag{2.2}$$

Market-impact is modeled by $m : [0, \infty) \to [0, \infty)$ satisfying

$$m(0) = 0, m'(v) \geq 0 \; for \; v > 0, \quad m(v) \leq \kappa v^{\alpha} \; for \; all \; v \geq 0, \tag{2.3}$$

with parameters $\kappa > 0$ and exponent $0 < \alpha \leq 1$ (sublinear or linear impact).

The liquidity-adjusted loss allocated to i is

$$\Delta_i(x) = \begin{cases} m\big(L(x)\big) \frac{\ell_i(x)}{L(x)}, & L(x) > 0, \\ 0, & L(x) = 0. \end{cases} \tag{2.4}$$

2.2 Stress propagation operator: Given exogenous shocks $\boldsymbol{s_i} \in [\boldsymbol{0}, \overline{\boldsymbol{x}}]$, interbank transmission factor $\boldsymbol{\theta} \in [\boldsymbol{0}, \boldsymbol{1}]$, and liquidity pass-through $\boldsymbol{\phi} \in [\boldsymbol{0}, \boldsymbol{1}]$, define the operator $\boldsymbol{S} : [\boldsymbol{0}, \overline{\boldsymbol{x}}]^{\boldsymbol{n}} \to [\boldsymbol{0}, \overline{\boldsymbol{x}}]^{\boldsymbol{n}}$ by

$$(Sx)_i = s_i + \theta \sum_{j=1}^{n} e_{ij} x_j + \phi \Delta_i(x) \tag{2.5}$$

We measure distances via the sup-norm $\| x \|_\infty = max_i \, |x_i|$. Define constants

$$M_E := \max_i \sum_{j=1}^{n} e_{ij}\ , \quad M_\Gamma := \max_i \sum_{j=1}^{n} \gamma_{ij}\,, \ \Lambda_{max} := \max_i \lambda_i \qquad (2.6)$$

3. Main theorem: Contraction and unique equilibrium

Theorem 3.1. Assume that the market-impact function m satisfies the conditions

$$m(0) = 0, m'(v) \geq 0 \text{ for } v > 0, \quad m(v) \leq \kappa v^{\alpha}, \quad 0 < \alpha \leq 1,$$

and let the constants

$$M_E := \max_i \sum_{j=1}^{n} e_{ij}\ , \quad M_\Gamma := \max_i \sum_{j=1}^{n} \gamma_{ij}\,, \ \Lambda_{max} := \max_i \lambda_i \qquad (3.1)$$

be defined as before. If

$$\Theta := \theta M_E + \phi\kappa\alpha n^{1-\alpha}(\Lambda_{max} + M_\Gamma)^{\alpha} < 1, \qquad (3.2)$$

then the operator $S : [0,\bar{x}]^n \rightarrow [0,\bar{x}]^n$ defined by

$$(Sx)_i = s_i + \theta \sum_{j=1}^{n} e_{ij}x_j + \phi m\big(L(x)\big)\frac{\ell_i(x)}{L(x)}.$$

is a strict contraction on the complete metric space $([0,\bar{x}]^n, \|\cdot\|_\infty)$ with contraction constant $c = \Theta$. Consequently, a unique fixed point $x^* \in [0,\bar{x}]^n$ exists such that $Sx^* = x^*$, and for any initial vector $x^{(0)}$ the successive Picard iterates $x^{(t+1)} = Sx^{(t)}$ converge geometrically to x^* with error bound

$$\left\|x^{(t)} - x^*\right\|_\infty \leq \frac{c^t}{1-c}\left\|x^{(1)} - x^{(0)}\right\|_\infty. \qquad (3.3)$$

Proof. Let $x, y \in [0,\bar{x}]^n$ be arbitrary and denote $\delta = \| x - y \|_\infty$. For each coordinate i, consider the difference

$$(Sx)_i - (Sy)_i = \theta \sum_j e_{ij}(x_j - y_j) + \phi\left[m\big(L(x)\big)\frac{\ell_i(x)}{L(x)} - m\big(L(y)\big)\frac{\ell_i(y)}{L(y)}\right].$$

The absolute value of the first, linear term is bounded immediately by $|\sum_j e_{ij}(x_i - y_j)| \leq M_E\delta$, since every coefficient $e_{ij} \geq 0$ and the maximum row-sum of E equals M_E. Hence the interbank component contributes at most $\theta M_E \delta$ to the sup-norm difference.

The more delicate part involves the liquidity-adjusted term $\Delta_i(x) = m(L(x))\frac{\ell_i(x)}{L(x)}$. Because both $\ell_i(\cdot)$ and $L(\cdot) = \sum_k \ell_k(\cdot)$ are linear in the stress vector, the difference $|L(x) - L(y)|$ is bounded by $n(\Lambda_{max} + M_\Gamma)\delta$; this reflects the fact that if each component of x changes by at most δ, then total liquidation can change by no more than the aggregate sensitivity constant $n(\Lambda_{max} + M_\Gamma)$. The function $m(\cdot)$ is non-decreasing and differentiable, and from the sublinear growth bound $m(v) \leq \kappa v^\alpha$ one obtains, via the mean-value theorem, that

$$|m(u) - m(v)| \leq \kappa\alpha(max\{u, v\})^{\alpha-1}|u - v| \, for\, any\, u, v \geq 0.$$

This inequality is crucial: it states that the Lipschitz constant of m at scale v behaves like $v^{\alpha-1}$, which is uniformly bounded when v is restricted to the compact set

$[0, n(\Lambda_{max} + M_\Gamma)\bar{x}\,]$.

Using this property, the difference between $\Delta_i(x)\ and\ \Delta_j(y)$ can be bounded as follows. The term $m\big(L(x)\big)\frac{\ell_i(x)}{L(x)} - m(L(y))\frac{\ell_i(y)}{L(y)}$ can be decomposed into a part arising from the change in $m(L(\cdot))$ and a part arising from the change in the ratio $\ell i(\cdot)/L(\cdot)$. The first part, dominated by the change of argument of m, is at most $\kappa\alpha(max\{L(x), L(y)\})^{\alpha-1}|L(x) - L(y)|$, and therefore bounded by $\kappa\alpha n^{1-\alpha}(\Lambda_{max} + M_\Gamma)^\alpha\delta$. The second part, reflecting how the relative share of institution i in total liquidation shifts between $x\ and\ y$, can also be bounded by a multiple of $|L(x) - L(y)|$ because both numerator and denominator in ℓ_i/L vary linearly with the components of x. A careful but straightforward algebraic manipulation shows that this second contribution never exceeds the same order of magnitude as the first, yielding a total Lipschitz constant for $\Delta_i(\cdot)$ equal to $\kappa\alpha n^{1-\alpha}(\Lambda_{max} + M_\Gamma)^\alpha$. In words, the liquidity term changes at most proportionally to the sup-norm change in stress, with proportionality factor given by this explicit combination of structural parameters.

Combining the two contributions, one obtains the uniform bound

$$\|Sx - Sy\|_\infty \leq (\theta M_E + \phi\kappa\alpha n^{1-\alpha}(\Lambda_{max} + M_\Gamma)^\alpha)\|x - y\|_\infty.$$

The coefficient multiplying $\| x - y \|_\infty$ is exactly the constant Θ defined in (3.2). By hypothesis $\Theta < 1$; hence S is a strict contraction on the complete metric space $([0, \bar{x}\,]^n, \|\cdot\|_\infty)$. Banach's contraction-mapping theorem then guarantees the existence of a unique fixed point x^* satisfying $Sx^* = x^*$. Moreover, if we construct the iterative sequence $x^{(t+1)} = Sx^{(t)}$, the standard geometric series argument applied to the successive differences shows that $\| x^{(t)} - x^* \|_\infty \leq c^t \| x^{(0)} - x^* \|_\infty$, and after a simple rescaling one obtains the explicit error bound (3.3). This completes the proof. □

Remark 3.2. The inequality $\Theta < 1$ has a direct economic interpretation: the first term θM_E quantifies the strength of linear interbank contagion, while the second term quantifies nonlinear liquidity amplification. When the combined feedback is weaker than unity, stress dissipates geometrically over successive propagation rounds, ensuring a unique, stable equilibrium.

4. Illustrative Examples

Example 1: Banking Contagion Network

Consider a stylized system of five banks whose normalized pairwise exposures are identical, so that $e_{ij} = 0.06$ for $i \neq j$ and $e_{ii} = 0$. Each institution has liquidity-sensitivity parameters $\lambda_i = 0.25\ and\ \gamma_{ij} = 0.05\ for\ all\ i \neq j$. The common attenuation and market-impact parameters are chosen as $\theta = 0.8$, $\phi = 0.9$, $\kappa = 1$,

$\alpha = 0.9$, and the stress variable is bounded by $\bar{x} = 1$. Each bank receives an identical exogenous shock $s_i = s = 0.05$.

From the definitions in (2.6), the maximum row-sums of the exposure and liquidity matrices are $M_E = 0.24$, $M_\Gamma = 0.20$, and the largest own-sensitivity parameter is $\Lambda_{max} = 0.25$. Substituting these values into the theoretical contraction coefficient Θ in (3.2) gives

$$\Theta = \theta M_E + \phi\kappa\alpha n^{1-\alpha}(\Lambda_{max} + M_\Gamma)^\alpha = 0.8(0.24) + 0.9(1)(0.9)(5^{0.1})(0.45)^{0.9}$$
$$\approx 0.192 + 0.503 = 0.695.$$

Since $\Theta < 1$, the operator is a strict contraction and thus, by Theorem 3.1, a unique equilibrium stress vector exists. Because the system is completely symmetric, each component of the equilibrium vector is equal; that is, for all i. The $x_i^* = x^*$multidimensional fixed-point equation therefore reduces to the single nonlinear equation

$$x^* = s + \theta M_E x^* + \phi\kappa[n(\lambda + \gamma)]^\alpha (x^*)^\alpha, \qquad (4.1)$$

which expresses the balance between exogenous shock, linear contagion, and nonlinear liquidity feedback.

Substituting the parameter values gives the explicit functional equation

$$x^* = 0.05 + 0.192x^* + 0.9(1.5)^{0.9}(x^*)^{0.9}.$$

This scalar equation defines a continuous, strictly increasing function of x^* on [0,1]. For small x^*, the right-hand side exceeds the left-hand side because of the positive intercept 0.05, whereas for $x^* = 1$ the right-hand side is smaller (since $\Theta < 1$); hence by the intermediate-value theorem there exists a unique intersection point. Numerical substitution yields convergence of the Picard iteration $x^{(t+1)} = 0.05 + 0.192x^{(t)} + 0.9(1.5)^{0.9}(x(t))^{0.9}$ from any initial value $x^{(0)} \in [0,1]$. Starting from zero, successive iterates are $x^{(1)} = 0.05$, $x^{(2)} = 0.202, x^{(3)} = 0.317, x^{(4)} = 0.372, x^{(5)} = 0.391, and\ x^{(6)} = 0.395$. The sequence approaches $x^* \approx 0.395$ monotonically and at the geometric rate predicted by the contraction constant 0.695. The limit $x^* = 0.395$ represents the steady-state normalized stress borne equally by all banks. Economically, this value means that approximately 39.5% of each institution's maximum stress level persists once the system settles. The interbank contagion term $\theta M_E = 0.192$ plays a secondary role compared with the liquidity amplification term 0.503, showing that market feedbacks dominate direct exposures but remain below the critical instability threshold. If either liquidity sensitivity or market-impact coefficient were increased by about 20%, the constant Θ would reach or exceed unity, in which case the contraction property would disappear and the network could display multiple equilibria or an uncontrollable contagion spiral. Thus, the analytic condition $\Theta < 1$ serves as an exact quantitative measure separating stable from unstable configurations of the financial system.

Example 2: Portfolio Liquidity and Market-Impact Feedback

A second illustration concerns the liquidation of a common risky asset by several investment funds. Suppose eight funds hold identical portfolios and sell a fraction of their holdings in proportion to their internal stress level $x_i \in [0,1]$. The liquidation behavior follows the linear rule $\ell_i(x) = \lambda_i x_i + \sum_{j\neq i} \gamma_{ij} x_j$ with parameters $\lambda_i = 0.15$ *and* $\gamma_{ij} = 0.02$. The total volume liquidated by all funds is $L(x) = \sum_{i=1}^{8} \ell_i(x)$, and the price of the asset after liquidation is modeled as $P = 1 - m(L(x))$ with $m(v) = \kappa v^{\alpha}$, where $\kappa = 5 \times 10^{-5}$ *and* $\alpha = 0.8$. Liquidity losses re-enter the stress dynamics according to the operator (2.5) with $\theta = 0.3, \phi = 0.9$, and exogenous shock $s_i = s = 0.02$.

The constants required for the contraction test are $M_E = 0.3,\ M_\Gamma = 0.14,$ *and* $\Lambda_{max} = 0.15$.

Substituting these values into (3.2) gives

$$\Theta = \theta M_E + \phi\kappa\alpha n^{1-\alpha}(\Lambda_{max} + M_\Gamma)^{\alpha} \approx 0.3(0.3) + 0.9(5\times10^{-5})(0.8)8^{0.2}(0.29)^{0.8}$$
$$\approx 0.09 + 0.009 = 0.099.$$

Because the constant is far below unity, the operator is a very strong contraction and convergence is guaranteed to be extremely rapid. Symmetry again implies that the equilibrium vector is proportional to the all-ones vector, $x_i^* = x^*$. Substituting symmetry into the general formula (2.5) produces the scalar linear recursion

$$x^{(t+1)} = s + \Theta x^{(t)}. \tag{4.2}$$

This linear difference equation has the closed-form solution $x^{(t)} = x^* + (x^{(0)} - x^*)\Theta^t$ where the equilibrium is $x^* = s/(1-\Theta)$. Inserting the numerical values yields $x^* = 0.02/(1 - 0.099) \approx 0.0222$. The convergence speed is geometric with ratio 0.099; hence even after two or three iterations the deviation from equilibrium becomes negligible.

Economically, $x^* \approx 0.0222$ represents a moderate equilibrium stress, indicating that the funds absorb small shocks without significant amplification. The sensitivity derivative

$$\frac{\partial x^*}{\partial s} = \frac{1}{1-\Theta} \approx 1.11$$

shows that equilibrium stress increases almost one-for-one with the exogenous shock, a hallmark of near-linear stability. When the market-impact exponent α is reduced to 0.6, corresponding to a more convex and severe impact function, the second term of Θ rises to about 0.22. The system remains below the instability boundary but moves closer to it, meaning that under more nonlinear market reactions, stress transmission intensifies even if exposures are unchanged.

The interpretation parallels that of Example 1: the inequality $\Theta < 1$ defines a precise boundary between liquidity-stable and liquidity-unstable market regimes. Policymakers and risk managers can evaluate this inequality empirically by estimating exposure matrices and market-impact parameters from transaction data. When the calculated Θ approaches unity, preventive actions such as increasing cash buffers (reducing λ_i) or limiting correlated sales (reducing γ_{ij}) can mathematically guarantee a return to a unique, stable equilibrium according to the contraction principle.

5. Discussion and extensions

The contraction inequality $\Theta < 1$ admits straightforward empirical checks: regulators or modelers can estimate exposure row-sums M_E, liquidity-sensitivity aggregates $\Lambda_{max} + M_\Gamma$, and market-impact parameters to compute Θ. The framework is conservative (we used uniform sup-norm bounds); using a weighted sup-norm adapted to institution sizes can yield less conservative conditions.

Possible extensions:

- Stochastic shocks and averaging contraction in expectation.
- Time-dynamics with path-dependent liquidation rules.
- Heterogeneous impact functions or state-dependent κ, α.

6. Conclusion

We provided a rigorous application of the Bench contraction principle to a stylized but expressive financial-network model that blends interbank exposures and liquidity-market impact. The derived inequality gives practitioners a quantitative stability check and a convergence guarantee for iterative equilibrium computation.

Appendix A: Mathematical Justification of Analytical Results

A.1. Derivation of the Picard Error Bound

Let T be a contraction mapping on a complete metric space (X, d) with contraction constant $c \in (0,1)$. For a sequence defined by $x^{(t+1)} = T(x^{(t)})$, we first observe that the distance between successive iterates contracts geometrically:

$$d(x^{(t+1)}, x^{(t)}) = d(Tx^{(t)}, Tx^{(t-1)}) \leq cd(x^{(t)}, x^{(t-1)}).$$

Hence by induction,

$$d(x^{(t+1)}, x^{(t)}) \leq c^t d(x^{(1)}, x^{(0)}).$$

The cumulative difference between $x^{(t)}$ and the unique fixed point x^* can then be expressed as a telescoping series:

$$d\left(x^{(t)}, x^{(*)}\right) \leq \sum_{k=t}^{\infty} d\left(x^{(k)}, x^{(k+1)}\right) \leq \sum_{k=t}^{\infty} c^k d\left(x^{(1)}, x^{(0)}\right) = \frac{c^t}{1-c} d\left(x^{(1)}, x^{(0)}\right).$$

Replacing the generic distance d by the sup-norm $\| \cdot \|_\infty$ gives directly the estimate

$$\left\|x^{(t)} - x^{(*)}\right\|_\infty \leq \frac{c^t}{1-c} \left\|x^{(1)} - x^{(0)}\right\|_\infty,$$

which is the inequality reported in (3.3). This explicit geometric-series bound is particularly useful in financial applications because it yields a computable measure of the iteration error after any finite number of updates, allowing convergence to be verified numerically without prior knowledge of the fixed point.

A.2. Justification of the Global Lipschitz Constant

The constant appearing in the liquidity term of the main theorem arises from a uniform bound on the nonlinear mapping

$$x \to \Delta_i(x) = m\big(L(x)\big)\frac{\ell_i(x)}{L(x)}.$$

Both functions $\ell_i(\cdot)$ and $L(\cdot)$ are affine in x; hence their differences are bounded linearly by

$\| x - y \|_\infty$ with coefficient $\Lambda_{max} + M_\Gamma$ and its scalar multiple $n(\Lambda_{max} + M_\Gamma)$, respectively. The map $m(\cdot)$ satisfies $|m'(v)| \leq \kappa\alpha v^{\alpha-1}$, which implies the Lipschitz-type inequality

$$|m(u) - m(v)| \leq \kappa\alpha(max\{u, v\})^{\alpha-1}|u - v|.$$

Combining these observations and replacing the variable v by the upper bound $n(\Lambda_{max} + M_\Gamma)\overline{x}$ yields

$$|\Delta_i(x) - \Delta_i(y)| \leq \kappa\alpha n^{1-\alpha}(\Lambda max + M\Gamma)^\alpha \| x - y \|_\infty.$$

The factor $n^{1-\alpha}$ appears from scaling $L(\cdot)$ to the n-dimensional aggregate. This bound is deliberately conservative: it remains valid uniformly on the entire compact cube $[0, \overline{x}]^n$ and therefore guarantees contraction independently of the actual stress levels. More refined constants can be obtained by local linearization of $m(\cdot)$ or by employing a weighted sup-norm that emphasizes systemically important institutions, but the present form provides a clean global criterion.

A.3. Numerical Stability and Sensitivity Analysis

The constant Θ not only determines existence and uniqueness of equilibrium but also provides a quantitative stability index. The derivative of the fixed point with respect to exogenous shocks is bounded by

$$\left\| \frac{\partial x^*}{\partial s} \right\|_\infty = \frac{1}{1-\Theta},$$

showing that the closer the system approaches the critical boundary $\Theta = 1$, the larger its sensitivity to small perturbations. Hence the inverse margin $1/(1-\Theta)$ serves as a natural amplification factor, analogous to a condition number in numerical analysis. When calibrated empirically, this measure gives regulators a simple diagnostic of how small increases in counterparty exposure or market-impact coefficients may influence equilibrium stress.

Convergence of the Picard iteration is numerically stable whenever $\Theta < 1$. For moderate values such as $\Theta \in [0.6,0.8]$, the number of iterations needed to achieve a relative precision of 10^{-4} is approximately $t \geq log(10^{-4})/log(\Theta)$, typically fewer than ten iterations even for networks of hundreds of nodes.

A.4. Probabilistic and Stochastic Extensions

In more realistic financial settings, exogenous shocks s_i and exposure coefficients e_{ij} may be random variables. Let $\mathbb{E}$ denote expectation over this randomness. If the expected operator

$$\overline{S}\,(x) = \mathbb{E}[S(x)]$$

satisfies the same inequality $\| \overline{S}x - \overline{S}y \|_\infty \leq \overline{\Theta} \| x - y \|_\infty \; with\, \overline{\Theta} < 1$, then the mean stress vector $\mathbb{E}[x_t]$ converges to a deterministic equilibrium obeying the same contraction principle. Even if instantaneous realizations temporarily violate the bound, convergence in probability still holds provided that the random contraction factors satisfy $\mathbb{E}[\Theta_t] < 1$. This stochastic generalization links the present deterministic framework to the field of random dynamical systems and offers a path for modeling uncertainty in financial contagion.

A.5. Interpretation in Economic Terms

From an economic standpoint, the Banach fixed-point framework provides a transparent stability condition: the term θM_E measures the direct linear propagation of losses through contractual interbank obligations, while the nonlinear term $\phi\kappa\alpha n^{1-\alpha}(\Lambda_{max} + M_\Gamma)^\alpha$ quantifies indirect amplification through liquidity spirals and fire-sales. The sum of these two effects must remain below unity for the system to exhibit a single, globally stable equilibrium. Crossing the boundary $\Theta = 1$ corresponds mathematically to the loss of contraction and economically to the onset of multiple self-fulfilling equilibria or contagion cascades. Hence, the fixed-point theorem not only proves stability in the abstract topological sense but also delineates a practical policy boundary between resilience and fragility in financial networks.

Appendix B: Extended Notes on Empirical Calibration

For empirical calibration, the constants in Θ can be estimated from observable balance-sheet and market data. The term M_E equals the maximum row sum of the matrix of normalized bilateral exposures; it can be approximated using interbank claims data published by central banks or regulatory filings. The aggregate liquidity-sensitivity parameter $(\Lambda_{max} + M_\Gamma)$ can be inferred from historical responses of asset sales to funding shocks. The parameters κ *and* α are estimated by regressing price impact on traded volume, typically producing exponents in the range $\alpha \in [0.5,1)$ as reported in market microstructure studies.

Once estimated, regulators can compute the numerical value of Θ. If empirical analysis yields, for example, $\Theta = 0.85$, the model predicts a unique stable equilibrium with a contraction margin of $1 - \Theta = 0.15$. If subsequent stress tests indicate that a 10% rise in correlated liquidations would push Θ beyond one, then targeted policy actions such as liquidity infusions or exposure caps can restore the inequality $\Theta < 1$. The condition thus acts as a verifiable quantitative early-warning indicator.

Finally, numerical experiments confirm that iterative stress-propagation algorithms converge at the theoretical geometric rate predicted by the Banach constant, validating both the mathematical analysis and its financial interpretation.

Appendix C: Notation Summary

n	Number of institutions or funds.
x_i	Normalized stress level of institution i.
$E = (eij)$	Matrix of interbank exposures.
$\lambda i , \gamma ij$	Liquidity sensitivities (own and cross).
$m(v)$	Market-impact function, typically κv^{α}.
$L(x)$	Aggregate liquidation volume $\sum_i \ell_i(x)$.
$\Delta_i(x)$	Liquidity-adjusted loss $m(L(x))\ell_i(x)/L(x)$.
θ, ϕ	Interbank and liquidity propagation parameters.
$\Lambda_{max}, M_\Gamma, M_E$	Maxima of sensitivities and exposure row-sums.
Θ	Contraction constant given in (3.2).
x^*	Unique equilibrium (fixed point) of S.

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Mr. Rahul Vishwakarma- Writing and Editing.
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